

# *Excursions through KK modes*

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## **Abstract**

In this article we study Kaluza-Klein (KK) dimensional reduction of massive Abelian gauge theories with charged matter fields on a circle. Since local gauge transformations change position dependence of the charged fields, the decomposition of the charged matter fields into KK modes is gauge dependent. While whole KK mass spectrum is independent of the gauge choice, the mode number depends on the gauge. The masses of the KK modes also depend on the field value of the zero-mode of the extra dimensional component of the gauge field. In particular, one of the KK modes in the KK tower of each massless 5D charged field becomes massless at particular values of the extra-dimensional component of the gauge field. When the extra-dimensional component of the gauge field is identified with the inflaton, this structure leads to recursive cosmological particle productions.

# 1 Introduction

Local gauge transformations change the position dependence of charged matter fields. As a result, when compactified on a circle, the decompositions of charged matter fields into Kaluza-Klein (KK) modes depend on the gauge [1]. In this article, we elaborate this point in a KK compactification of massive Abelian gauge theory with charged matter fields. After the KK dimensional reduction to 4D, while whole KK mass spectrum is independent of the gauge, the mode number depends on the gauge. The masses of the KK modes of the charged matter fields also depend on the field value of the zero-mode of the Extra-Dimensional Component of the Gauge Field (EDCGF for short in the following). In particular, one mode in the KK tower in each massless 5D charged field becomes massless at particular field values of the zero-mode of EDCGF. Fields which become massless at particular field values of inflaton lead to cosmological particle productions which may have interesting observational consequences [2, 3, 4, 5, 6]. In the meantime, the zero-mode of EDCGF plays the role of inflaton in models of inflation [7, 8]. In these models, the inflaton makes large field (trans-Planckian) excursion to explain the observed anisotropy of the Cosmic Microwave Background (CMB) [9, 10]. If we consider our model in this context, the inflaton passes through the points at which the KK mode becomes massless during its excursion, explaining the title of this article. The mechanism explained in this article is general in models of inflation based on higher-dimensional gauge theories as it is at work whenever a light charged matter field is present in the model.

## 2 KK modes of matter fields in 5D Abelian gauge theory

To present the basic mechanism in the simplest example, we consider the following 5D action of massive gauge field coupled to a massless scalar field:

$$S_5 = \int d^5x \left[ -\frac{1}{4} F_{MN}(x) F^{MN}(x) - V(\mathcal{A}_M(x)) + D_M \chi^\dagger(x) D^M \chi(x) \right],$$
$$(M, N = 0, 1, 2, 3, 5), \quad (1)$$

where

$$\mathcal{A}_M(x) = A_M(x) - \frac{i}{g_5} e^{i\theta(x)} \partial_M e^{-i\theta(x)}, \quad (2)$$

and  $\theta(x)$  is the Stueckelberg field. Note that the Stueckelberg field is an angular variable as seen from (2):

$$\theta(x) \sim \theta(x) + 2\pi w, \quad (3)$$

with  $w$  being an integer and  $\sim$  denotes the identification. The field strength of the Abelian gauge field is given as

$$F_{MN}(x) = \partial_M A_N(x) - \partial_N A_M(x). \quad (4)$$

We assume the minimal coupling of the field  $\chi(x)$  to the gauge field:

$$D_M \chi(x) = \partial_M \chi(x) - ig_5 A_M(x) \chi(x). \quad (5)$$

The potential for the vector field  $V(\mathcal{A}_M)$  may be expanded in power series in  $\mathcal{A}_M \mathcal{A}^M$ . We take  $V(\mathcal{A}_M) = m^2 \mathcal{A}_M \mathcal{A}^M / 2$  to make the following discussions simple, but higher order terms can be easily included. The action (1) is invariant under the local  $U(1)$  gauge transformation generated by  $U(x) = \exp[i\Theta(x)]$ :

$$\begin{aligned} A_M(x) &\rightarrow \tilde{A}_M(x) = A_M(x) + \frac{i}{g_5} U(x) \partial_M U^{-1}(x), \\ \theta(x) &\rightarrow \tilde{\theta}(x) = \theta(x) + \Theta(x), \\ \chi(x) &\rightarrow \tilde{\chi}(x) = U(x) \chi(x), \\ \chi^\dagger(x) &\rightarrow \tilde{\chi}^\dagger(x) = \chi^\dagger(x) U^{-1}(x). \end{aligned} \quad (6)$$

Here, we indicated by  $\sim$  that they are the fields after the gauge transformation.

We are interested in the KK dimensional reduction on a circle with radius  $L_5$  in the 5-th direction. We impose periodic boundary conditions on the fields  $\chi(x)$ ,  $\chi^\dagger(x)$  and  $A_M(x)$ :<sup>1</sup>

$$\psi(x^5 + 2\pi L_5) = \psi(x^5), \quad \psi(x) = A_M(x), \chi(x), \chi^\dagger(x). \quad (7)$$

However, the Stueckelberg field can have a winding mode as it is an angular variable as in (3):

$$\theta(x^5 + 2\pi L_5) = \theta(x^5) + 2\pi w \sim \theta(x), \quad (w : \text{integer}). \quad (8)$$

To study KK dimensional reduction, we decompose the field into Fourier modes:

$$A_M(x, x^5) = \frac{1}{\sqrt{2\pi L_5}} \sum_{n=-\infty}^{\infty} A_M^{(n)}(x) e^{i \frac{n}{L_5} x^5}, \quad (9)$$

$$\theta(x, x^5) = \frac{w x^5}{L_5} + \frac{1}{\sqrt{2\pi L_5}} \sum_{n=-\infty}^{\infty} \theta_n(x) e^{i \frac{n}{L_5} x^5}, \quad (w : \text{integer}), \quad (10)$$

$$\chi(x, x^5) = \frac{1}{\sqrt{2\pi L_5}} \sum_{n=-\infty}^{\infty} \chi_n(x) e^{i \frac{n}{L_5} x^5}, \quad (11)$$

$$\chi^\dagger(x, x^5) = \frac{1}{\sqrt{2\pi L_5}} \sum_{n=-\infty}^{\infty} \chi_n^\dagger(x) e^{-i \frac{n}{L_5} x^5}. \quad (12)$$

From here we slightly change our notation:  $x^5$  is distinguished from  $x$  which collectively denotes 4D Minkowski coordinates. It is important to notice that the KK mode labels  $n$  for the charged fields  $\chi(x, x^5)$  and  $\chi^\dagger(x, x^5)$  in (11) and (12) are gauge dependent, since they can be shifted by the gauge transformation (6) with the gauge parameter

$$U(x, x^5) = e^{i\Theta(x, x^5)}, \quad \Theta(x, x^5) = \frac{x^5}{L_5}. \quad (13)$$

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<sup>1</sup>These fields can have boundary conditions twisted by the global part of the gauge transformation, but this case reduces to the periodic boundary conditions (7) by field redefinitions.

Note that the gauge transformation (13) does not change the periodic boundary conditions (7) and (8) on the fields and thus is a legitimate gauge transformation. The gauge transformation (13) shifts the Fourier modes including the zero-modes of the fields as

$$\chi_n(x) \rightarrow \tilde{\chi}_n(x) = \chi_{n-1}(x), \quad \chi_n^\dagger(x) \rightarrow \tilde{\chi}_n^\dagger(x) = \chi_{n-1}^\dagger(x), \quad (14)$$

$$A_5^{(0)}(x) \rightarrow \tilde{A}_5^{(0)}(x) = A_5^{(0)}(x) + \frac{1}{g_4 L_5}, \quad w \rightarrow \tilde{w} = w + 1. \quad (15)$$

In (15) we have introduced the four-dimensional gauge coupling  $g_4$  which is related to the 5D one as

$$g_4 = \frac{g_5}{\sqrt{2\pi L_5}}. \quad (16)$$

Some comments may be in order regarding (14). From (6) the gauge transformation (13) acts on the field  $\chi(x, x^5)$  as follows:

$$\chi(x, x^5) \rightarrow \tilde{\chi}(x, x^5) = e^{i \frac{x^5}{L_5}} \chi(x, x^5), \quad (17)$$

We write the Fourier decomposition of  $\tilde{\chi}$  as follows:

$$\tilde{\chi}(x, x^5) = \frac{1}{\sqrt{2\pi L_5}} \sum_{n=-\infty}^{\infty} \tilde{\chi}_n(x) e^{i \frac{n}{L_5} x^5}, \quad (18)$$

Then, through (17) the Fourier modes  $\chi_n(x)$  and  $\tilde{\chi}_n(x)$  are related as

$$\tilde{\chi}_n(x) = \chi_{n-1}(x). \quad (19)$$

As shown below, the 4D masses of the fields  $\chi_n(x)$  and  $\chi_n^\dagger(x)$  depend on the field value of  $A_5^{(0)}$ . Unless we specify the field value of  $A_5^{(0)}$ , we cannot tell which mode is the lightest, except which we wish to integrate out to obtain the low energy effective theory below the compactification energy scale  $L_5^{-1}$ . Therefore, we keep all the Fourier modes of  $\chi(x, x^5)$  and  $\chi^\dagger(x, x^5)$  in KK dimensional reduction, while keeping only the zero-modes for other fields.<sup>2</sup> Upon KK dimensional reduction, we obtain the following 4D classical action:

$$\begin{aligned} S_4 = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) + \frac{m^2}{2} \mathcal{A}_\mu(x) \mathcal{A}^\mu(x) \right. \\ \left. + \frac{1}{2} D_\mu \phi(x) D^\mu \phi(x) - \frac{m^2}{2} (\phi(x) - 2\pi f w)^2 \right. \\ \left. + \sum_{n=-\infty}^{\infty} \{ D_\mu \chi_n^\dagger(x) D^\mu \chi_n(x) + \chi_n^\dagger(x) g_4^2 (\phi(x) - 2\pi f n)^2 \chi_n(x) \} \right], \\ (\mu = 0, 1, 2, 3), \end{aligned} \quad (20)$$

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<sup>2</sup>The necessity of this generalized KK dimensional reduction including all the KK modes of the charged fields has been noted in [1].

where we have defined

$$2\pi f = \frac{1}{g_4 L_5}. \quad (21)$$

In (20), we have omitted the suffix 0 of the zero-modes, i.e. we have renamed the fields as  $A_\mu^{(0)}(x) \Rightarrow A_\mu(x)$ ,  $A_5^{(0)}(x) \Rightarrow \phi(x)$ ,  $\theta_0(x) \Rightarrow \theta(x)$ , and then we defined  $\mathcal{A}_\mu(x) = A_\mu(x) - ie^{i\theta(x)}\partial_\mu e^{-i\theta(x)}$ . The action is invariant under the discrete shift (14), (15) whose origin is the gauge symmetry (6):

$$\begin{aligned} \chi_n(x) &\rightarrow \tilde{\chi}_n(x) = \chi_{n-1}(x), & \chi_n^\dagger(x) &\rightarrow \tilde{\chi}_n^\dagger(x) = \chi_{n-1}^\dagger(x), \\ \phi(x) &\rightarrow \tilde{\phi}(x) = \phi(x) + 2\pi f, & w &\rightarrow \tilde{w} = w + 1. \end{aligned} \quad (22)$$

Note that after the KK dimensional reduction, only the transformation of the winding mode  $w$  of the 5D field  $\theta(x, x^5)$  remains.

Now, suppose that the field  $\phi(x)$  at first had a vacuum expectation value  $\langle\phi\rangle$ , then it makes an excursion by  $2\pi f$ :

$$\langle\phi\rangle \longrightarrow \langle\phi\rangle + 2\pi f. \quad (23)$$

In the above we have used the long right arrow to denote the field excursion. We observe

1. The potential energy  $V(\phi) = m^2 (\phi(x) - 2\pi f w)^2 / 2$  changes as

$$\begin{aligned} V(\langle\phi\rangle) &= \frac{m^2}{2} (\langle\phi\rangle - 2\pi f w)^2 \\ \longrightarrow V(\langle\phi\rangle + 2\pi f) &= \frac{m^2}{2} (\langle\phi\rangle - 2\pi f (w - 1))^2. \end{aligned} \quad (24)$$

2. The mass square of the fields  $\chi_n(x)$  and  $\chi_n^\dagger(x)$  flows as

$$g_4^2 (\langle\phi\rangle - 2\pi f n)^2 \longrightarrow g_4^2 (\langle\phi\rangle - 2\pi f (n - 1))^2, \quad (25)$$

As noted in the first point, the value of the potential energy  $V(\phi)$  changes after the excursion (23). Using the inverse of the gauge transformation (22), one can identify the field value  $\langle\phi\rangle + 2\pi f$  with the original value  $\langle\phi\rangle$ , but then the winding number  $w$  must be gauge transformed to  $w - 1$ . These mean that the system has undergone monodromy. Models which realizes large field inflation using monodromy have been extensively studied started from [11, 12]. From the second point above, we observe that while the mass of each KK mode of the charged scalar field shifts by the excursion (23), the spectrum as a whole comes back to the original one.

Note that when the 4D gauge coupling  $g_4 \lesssim 1$ ,  $2\pi f \gtrsim L_5^{-1}$  from (21). For applications to inflationary cosmology,  $L_5^{-1}$  should be larger than the Hubble scale at the time of inflation. Though such field excursion might look rather large, large field excursions are very common in inflation models, for example in large field inflation models inflaton makes trans-Planckian excursion. We will present an example of such models explicitly in section 3.

The interaction between the field  $\chi_n(x)$ ,  $\chi_n^\dagger(x)$  and  $\phi(x)$  in (20) is of the type studied in [2, 3, 4, 5, 6]. In these models, at some value(s) of the inflaton field, some field(s)

become(s) massless and cosmological particle productions follow, which may have observational consequences. In the current model, when the field  $\phi(x)$  passes through the field value  $\phi = 2\pi fn$  ( $n$ : integer), the modes  $\chi_n(x)$  and  $\chi_n^\dagger(x)$  become massless. That the field interval between the points at which the matter fields become massless are equally spaced reflects the geometry of the compactified space (circle), and it will be different for different compactified spaces. The mass spectrum flow (25) follows from the minimal coupling and it is not restricted to scalar matter fields. Similar mass spectrum flow appears in models with fermionic matter fields massless in 5D. As mentioned in the introduction, applications of cosmological particle productions from fields which become massless at particular field values of inflaton have been extensively studied following [2, 3, 4, 5, 6], while models of inflation in which the zero-mode of EDCGF plays the role of inflaton have been extensively investigated following [7, 8]. Our study shows that such cosmological particle productions are general feature of the inflation models based on higher-dimensional gauge theories which contain light charged fields, and should be examined whenever one considers such models.

### 3 Comparison of a simple model with observations

In this section we study a simple model which exhibits the resonant particle productions discussed in the previous section in light of recent CMB observations. The purpose here is illustrative rather than comprehensive. Some extensions of the model are discussed in the next section.

We consider a model in which the zero-mode  $\phi$  of EDCGF is identified with the inflaton. We first assume that the back-reactions of the resonant particle creations are small, and later examine the parameter region where this assumption is consistent. Under this assumption, the slow-roll parameters from the quadratic potential (24) are calculated as

$$\epsilon(\phi) := \frac{1}{2} \left( \frac{V'}{V} \right)^2 = \frac{2}{\phi^2}, \quad \eta(\phi) := \frac{V''}{V} = \frac{2}{\phi^2}. \quad (26)$$

Here, we work in the unit  $M_P = 1$ , where  $M_P := (8\pi G_N)^{-1/2}$  is the reduced Planck mass. The spectral index is given as

$$n_s \simeq 1 - 6\epsilon(\phi_*) + 2\eta(\phi_*), \quad (27)$$

where the subscript  $*$  refers to the value at the pivot scale  $0.002 \text{ Mpc}^{-1}$  of the Planck 2015 analysis [9]. The number of e-folds is given as

$$N(\phi) \simeq \int_{\phi_{end}}^{\phi} d\phi \frac{V}{V'} = \left[ \frac{\phi^2}{4} \right]_{\phi_{end}}^{\phi}, \quad (28)$$

where we have defined  $\phi_{end}$  by  $\epsilon(\phi_{end}) = 1$ . This gives

$$\phi_{end} = \sqrt{2}. \quad (29)$$

The scalar power spectrum is given by

$$P_s \simeq \frac{V(\phi_*)}{24\pi^2\epsilon(\phi_*)} = 2.2 \times 10^{-9}, \quad (30)$$

where the right hand side is the observed value [9]. The tensor-to-scalar ratio is given as

$$r_* \simeq 16\epsilon(\phi_*). \quad (31)$$

Now we choose  $N_* = 60$ , which gives

$$\phi_* \simeq 16, \quad (32)$$

$$r \simeq 0.13, \quad (33)$$

$$n_s \simeq 0.97, \quad (34)$$

$$m \simeq 6.0 \times 10^{-6}, \quad (35)$$

$$H_* \simeq 5.8 \times 10^{-5}. \quad (36)$$

While the chaotic inflation with quadratic potential is moderately disfavored according to the Planck 2015 analysis [10], it is possible to improve the fit to the data by a well-motivated modification of the potential [13]. Since our purpose here is to illustrate the main idea in a simple model, we proceed with the quadratic potential model.

There are also loop corrections to the effective potential of the inflaton. At one-loop, the contributions from the charged matter fields  $\chi$  and  $\chi^\dagger$  give [1]

$$V_{1-loop} = \frac{3}{\pi^2(2\pi L_5)^4} \left(1 - \cos \frac{\phi}{f}\right). \quad (37)$$

The effects of this term on CMB anisotropy have been studied in [14, 15]. Here, we restrict ourselves to the parameter region where these effects are small. This is the case when

$$V' \gg V'_{1-loop} \simeq \frac{3}{\pi^2 f (2\pi L_5)^4}. \quad (38)$$

Putting the field value  $14 \lesssim \phi \lesssim 16$  during the observable inflation and the inflaton mass (35) into (38), we obtain

$$\frac{1}{L_5} < g_4^{-1/3} \times 10^{-2}. \quad (39)$$

In order to justify the use of four-dimensional Einstein gravity, the Hubble expansion rate at the time of inflation should be smaller than the compactification energy scale  $1/L_5$ . From (36), this condition gives

$$\frac{1}{L_5} > 6 \times 10^{-5}. \quad (40)$$

In [5], the peak amplitude in the CMB power spectrum from a single isolated particle resonance due to the interaction between the inflaton and the charged fields  $\chi$ ,  $\chi^\dagger$  in the form in (20) was numerically obtained as

$$A_{IR} \simeq 2 \times g_4^{15/4} \times 10^{-6}, \quad (41)$$

(the factor 2 in (41) comes from the fact that  $\chi$  and  $\chi^\dagger$  are complex scalar fields). We assume that the contribution from a single particle resonance is smaller than the contributions of the inflaton vacuum fluctuations (30),  $P_s > A_{IR}$ . This condition gives

$$g_4 < 2 \times 10^{-1}. \quad (42)$$

The number of e-folds between the  $i$ -th particle resonance to  $i+1$ -th particle resonance is given as

$$N(\phi(k_i)) - N(\phi(k_{i+1})) = N(\phi(k_i)) - N(\phi(k_i) - 2\pi f) = \ln \frac{k_{i+1}}{k_i}. \quad (43)$$

During the observable inflation, the Hubble expansion rate does not change much. Therefore, we can safely take the part linear in  $2\pi f$  in the left hand side of (43). Then we obtain

$$k_{i+1} \simeq e^\Delta k_i, \quad (44)$$

where

$$\Delta = \frac{dN}{d\phi} 2\pi f = \frac{\phi}{2} 2\pi f. \quad (45)$$

Putting  $\phi \simeq \phi_* \simeq 16$ , we obtain

$$\Delta \sim \frac{8}{g_4 L_5}. \quad (46)$$

Then from (39) and (40) we obtain

$$g_4^{-1} \times 5 \times 10^{-4} < \Delta < g_4^{-4/3} \times 8 \times 10^{-2}. \quad (47)$$

For a reasonable value  $g_4 = 0.1$ , we have

$$5 \times 10^{-3} < \Delta < 2. \quad (48)$$

(48) mostly falls in the case of multiple bursts of particle production in [16]. Note that when  $\Delta \ll 1$ , it becomes harder to observationally distinguish each resonant particle production.

## 4 Note on models with more extra dimensions

Instead of the 5D model discussed in the previous section, we could have started with a gauge theory with more extra dimensions. In this case, we have multiple zero-modes of the extra-dimensional components of the gauge field. When the field space of the zero-mode of the gauge field is one-dimensional like in the 5D model studied in the previous section, the field  $\phi$  must pass through the point where the KK modes of the charged fields become massless if it travels more than  $2\pi f$ . On the other hand, if the field space of the zero-modes of the gauge field has more than two dimensions, the trajectory in the field space need not pass through the points where the KK modes of the charged



fields become massless. If one prefers to work in a model where the KK modes of the charged fields certainly become massless, one may better assume that the size of the 5-th dimension should be hierarchically larger than the other extra dimensions so that below the compactification energy scale of other extra dimensions the model effectively reduces to a 5D model. Another choice for constructing a model in which KK modes of charged field becomes light may be to increase the number density of the points in the field space where the KK modes of charged matter fields become massless, so that the trajectory in the field space of the zero-modes of the extra-dimensional components of the gauge field is likely to pass close to these points. Such models can be constructed as a simple extension of the model discussed in this article. For example, one can consider a product gauge group  $U(1)^r$  with sufficiently large integer  $r$  and arrange the model so that the vacuum expectation values of the zero-modes of the extra-dimensional components of the gauge field distribute uniformly and dense in the field space of the zero-modes. The zero-mode of EDCGF of one of the  $U(1)$  or one linear combination of  $U(1)$ 's may play the role of inflaton. Then one can assign the  $U(1)$  charges so that when the inflaton field value coincides with the expectation value of EDCGF of one  $U(1)$  the KK mode of some charged matter field(s) become(s) massless. One may further embed this model into a model with larger gauge group  $U(r)$  which is spontaneously broken to  $U(1)^r$ . A system similar to this case has been studied in [6] but in a T-dual picture, which we discuss in the next section.

## 5 T-dual picture

Some extension of the action (20) may arise as a low-energy effective field theory of a D-brane system. That system would have T-dual description in which the momentum numbers and the winding numbers are interchanged [17]. In the T-dual picture, the role of the zero-mode of EDCGF in the model discussed in this article will be played instead by the distance between different D-branes, and the Fourier mode number  $n$  will be replaced by a winding number, with the dual compactification radius  $\tilde{L}_5 = \alpha'/L_5$ . Note however that when  $L_5 \gg \alpha'^{1/2}$  the description based on the momentum modes is simpler, as in the dual picture the compactification radius is much smaller than the string scale and one has to deal with the full stringy description. In other words, two pictures are good descriptions in different regions of parameter space. Also note that T-dual picture is available only when the compactified space has a  $U(1)$  isometry. We can consider more general compactification of higher-dimensional gauge theories on a manifold without  $U(1)$  isometry. Keeping these points in mind, we observe that the example studied in this article is similar to the examples studied in [4, 6].

## 6 Discussions

In this article we studied KK dimensional reduction of higher-dimensional massive Abelian gauge theories with charged matter fields, and observed the appearance of massless fields at particular field values of the zero-mode of EDCGF. While in this article we took a simple model as an example, the appearance of massless fields at particular field values of the zero-mode of EDCGF is a general feature in inflation models based on higher-dimensional gauge theories, and should be examined further in such models. It will particularly be interesting to study observational consequences of the mechanism studied in this article in explicit cosmology models. It will also be interesting to study compactifications on different manifolds.

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